Bode Plot

Wednesday, September 20, 2017 8:06 AM

Draw the Bode plot of the following transfer function and find the magnitude at ω = 20, 50, 100 and 500 rad/sec.

$$T(s) = \frac{r_1\angle\theta_1 \; r_2\angle\theta_2}{r_3\angle\theta_3 \; r_4\angle\theta_4}$$

$$T(s) = 10^6 \frac{s(s+200)}{(s+10)(s+50)}$$

$$= K \cdot \frac{s\left[1+\frac{s}{200}\right]}{\left[1+\frac{s}{10}\right]\left[1+\frac{s}{50}\right]}$$

$\omega_1 = 10$ rad/sec

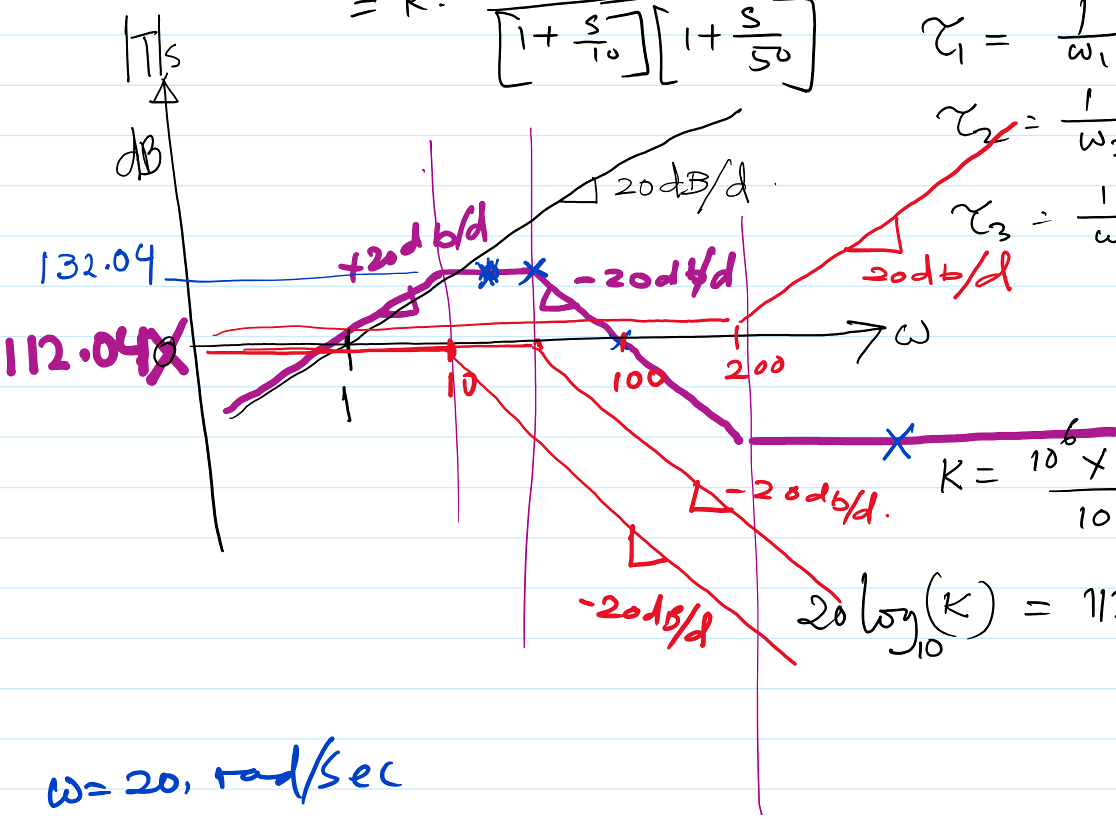
$\omega_2 = 50$ rad/sec

$\omega_3 = 200$ rad/se

$\tau_1 = \frac{1}{\omega_1} = \frac{1}{10}$ s

$\tau_2 = \frac{1}{\omega_2} = \frac{1}{50}$ s

$\tau_3 = \frac{1}{\omega_3} = \frac{1}{200}$ rad/sec



$$K = \frac{10^6 \times 200}{10 \times 50}$$

$$20\log_{10}(K) = 112.04\,dB$$

$\omega = 20$, rad/sec

$$T(s) = K \times \frac{s}{(1+s/10)} \simeq K \times \frac{s}{\left(\frac{s}{10}\right)} = \underline{\underline{K \times 10}}$$

$$= K \times 10$$

$$20\log|T(s)| = 20\log_{10} K + 20\log_{10} 10$$

$\omega = 50$ rad/sec,

$$T(s) = K \cdot \frac{s}{\left(1+\frac{s}{10}\right)} = K \times 10$$

$\omega = 100$ rad/Sec

$$s \qquad \simeq \frac{K\;\cancel{s}}{}$$

$\omega = 100 \text{ rad/sec}$

$$T(s) = K \cdot \frac{s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{50}\right)} \simeq \frac{K \cancel{s}}{\left(\frac{\cancel{s}}{10}\right)\left(\frac{s}{50}\right)}$$

$$= \frac{K \times 10 \times 50}{s}$$

100

$$|T(s)|_{dB} = 20\log_{10} K + 20\log_{10} 10 + 20\log_{10} 50 - 20\log_{10} \omega$$

$$= 112.04 + 20 + 34 - 40$$

$$= 126.04$$

$\omega = 500 \text{ rad/sec}$

$$T(s)|_{dB} = K \cdot \frac{s\left(1+\frac{s}{200}\right)}{\left(1+\frac{s}{10}\right)\left(1+s/50\right)} \simeq K \frac{\cancel{s} \times \frac{\cancel{s}}{200}}{\frac{\cancel{s}}{10} \times \frac{\cancel{s}}{50}}$$

$$= \frac{K \times 10 \times 50}{200}$$

$$T(s)|_{dB} = 20\log_{10} K + 20\log_{10} 10 + 20\log_{10} 50 - 20\log_{10} 200$$

$$= 120 \text{ db}$$

$$\boxed{\begin{aligned} x + jy &= r\angle\theta \\ r &= \sqrt{x^2+y^2} \\ \theta &= \tan^{-1}\frac{y}{x} \end{aligned}}$$

$K = |K|\angle 0$

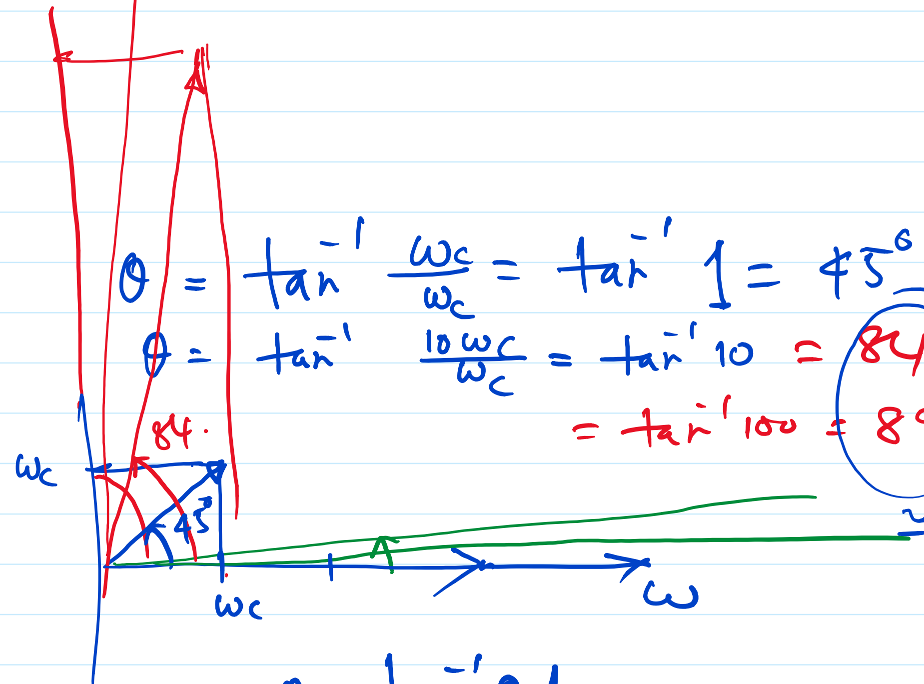
$s = j\omega = |\omega|\angle 90^\circ$

$$1 + \frac{s}{\omega_c} = \left|1 + \frac{j\omega}{\omega_c}\right| = \sqrt{1^2 + \left(\frac{\omega}{\omega_c}\right)^2}$$

$$1+\frac{s}{\omega_c} = \left|1 + j\frac{\omega}{\omega_c}\right| = \sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}$$

$$\approx \sqrt{\left(\frac{\omega}{\omega_c}\right)^2} \approx \frac{\omega}{\omega_c}$$

$$\angle 1+\frac{s}{\omega_c}\ \theta = \angle \tan^{-1}\frac{\omega}{\omega_c}$$



$\omega = \omega_c$ $\qquad \theta = \tan^{-1}\frac{\omega_c}{\omega_c} = \tan^{-1} 1 = 45^\circ$

$\omega = 10\,\omega_c$ $\qquad \theta = \tan^{-1}\frac{10\,\omega_c}{\omega_c} = \tan^{-1} 10 = 84$

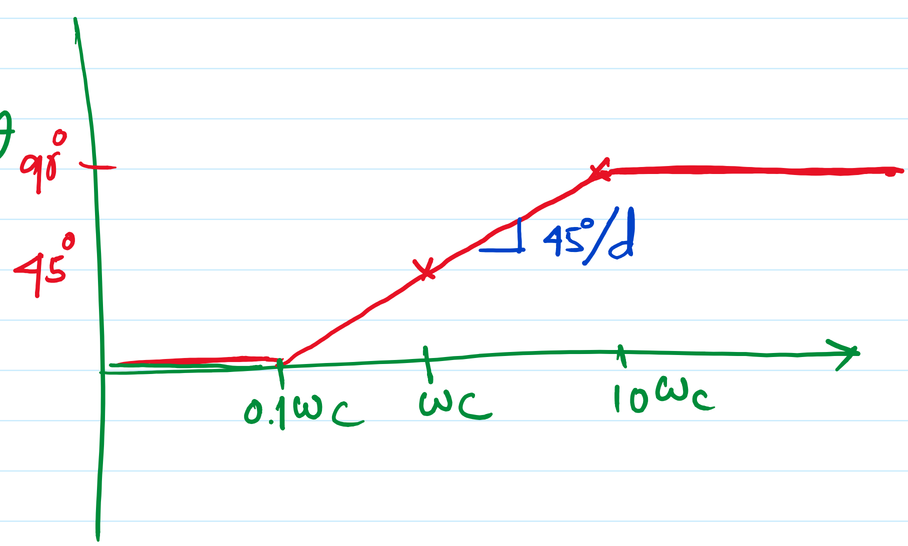
$\omega = 100\,\omega_c$ $\qquad = \tan^{-1} 100 = 89$

$\omega = 1000\,\omega_c$ $\qquad \approx 90^\circ$

$\omega = 0.1\,\omega_c$ $\qquad \theta = \tan^{-1} 0.1$

$\omega = 0.01\,\omega_c$ $\qquad \theta = \tan^{-1} 0.01$

$$\angle 1+\frac{s}{\omega_c} = \theta$$

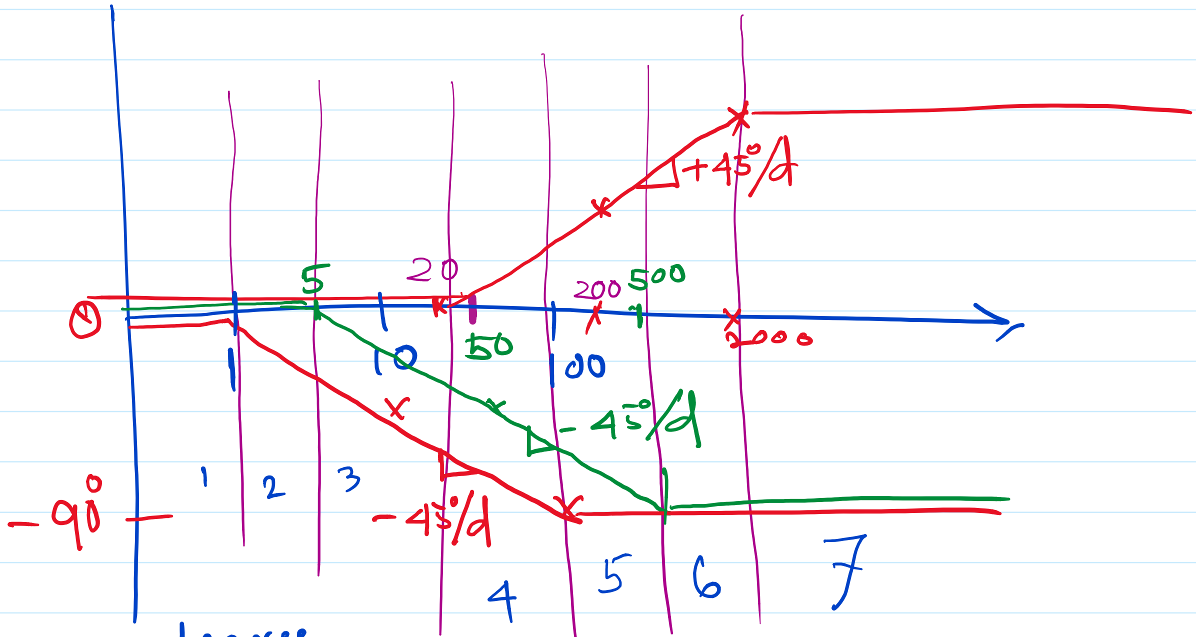


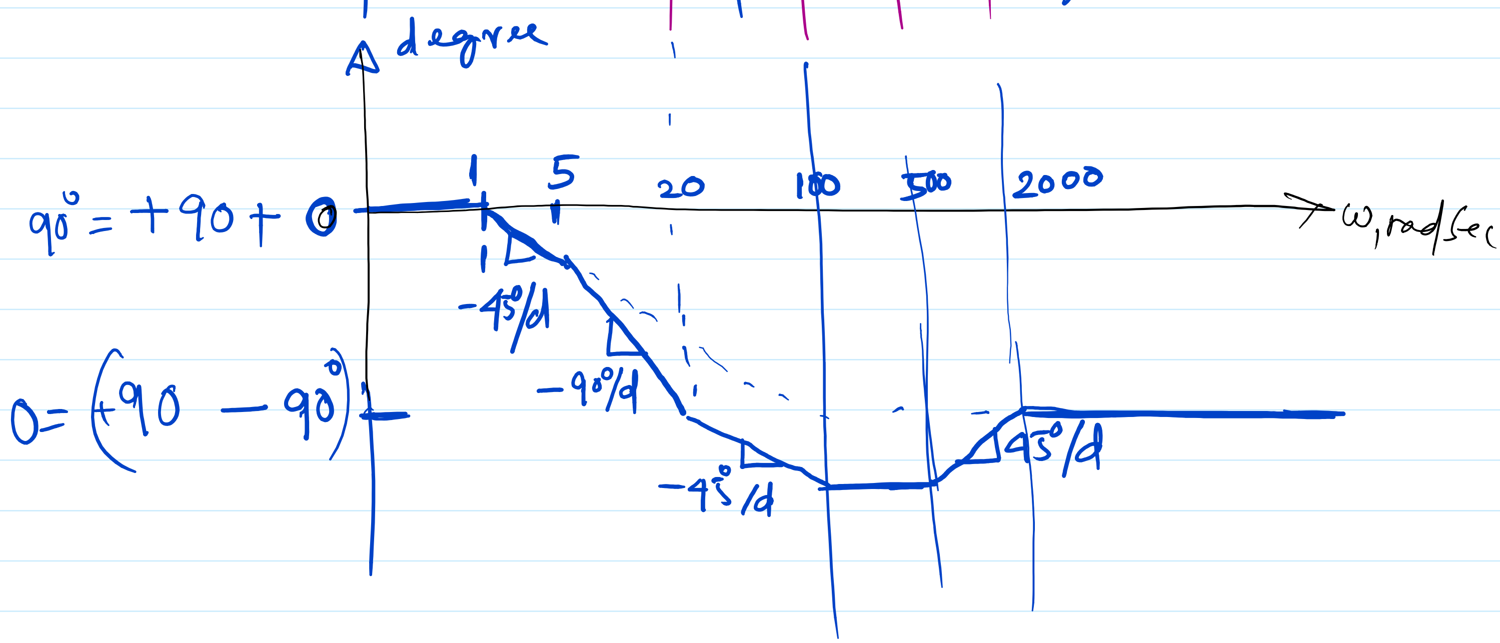
$Z = j\omega L$

$Z = \frac{-j}{\omega C}$

D$(-\theta)$ N$(+\theta)$

$\omega_c = (10, 50)\ (200.)$





$$\theta = 90 + \tan^{-1}\frac{\omega}{200} - \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{50}$$

$\omega = 20$ $\qquad\qquad$ $\theta_{20} = 10.47°$

$= 50$ $\qquad\qquad$ $\theta_{50} = -19.65$

$= 100$

$= 500$

$= 100$
$= 500$

$\theta_{50} = -17.63$
$\theta_{100} = -31.16$
$\theta_{500} = -14.95$